$$u(x,t) = \underbrace{\frac{\cos\frac{\Omega}{c}(l-x)}{\cos\frac{\Omega}{c}l}}_{U(x,\Omega)} \underbrace{U_G(\Omega)}_{\text{pobuda}} e^{j\Omega t}$$

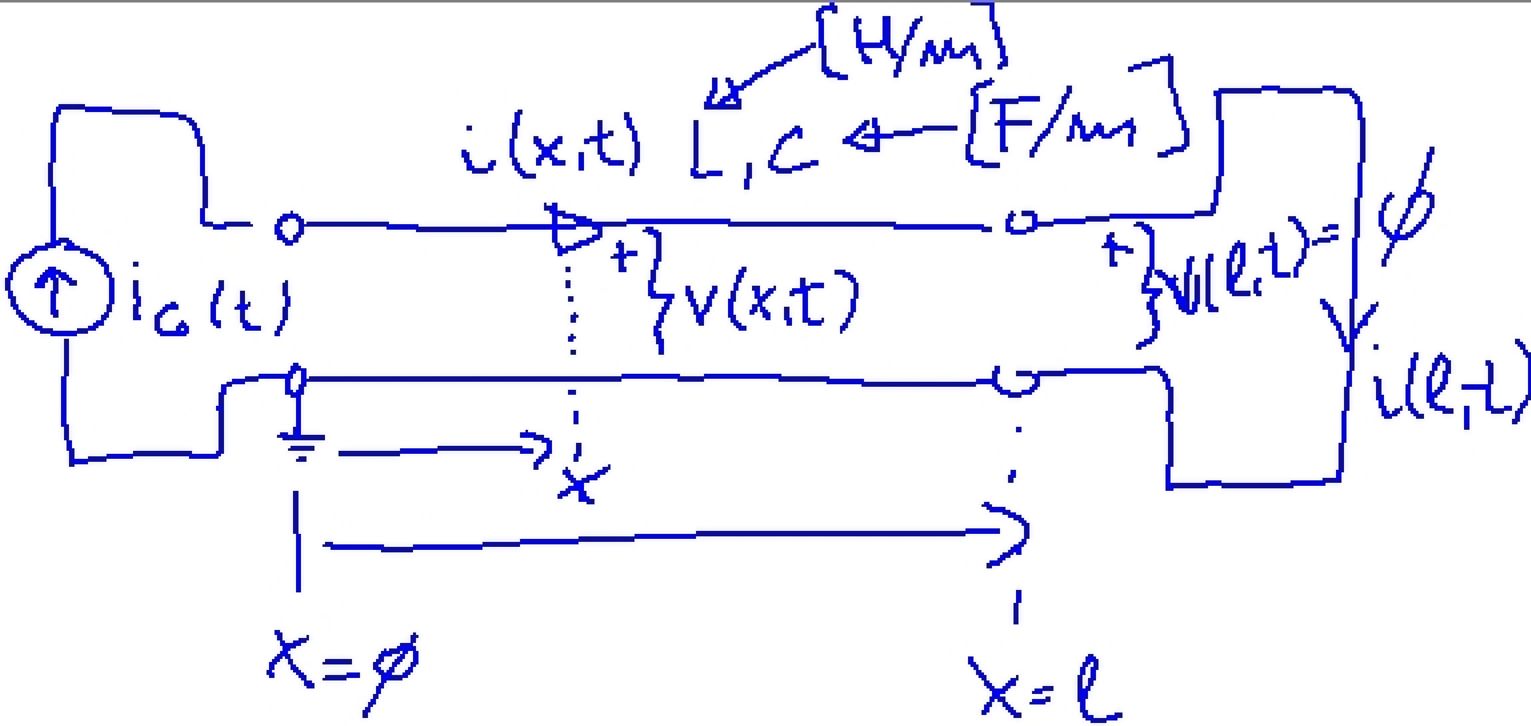
$$p(x,t) = \underbrace{\frac{\rho c}{A}}_{Z_0} \cdot j\,\frac{\sin\frac{\Omega}{c}(l-x)}{\cos\frac{\Omega}{c}l} \overbrace{U_G(\Omega) e^{j\Omega t}}$$

Sustav sa cijevi jednolikog presjeka A, l

bez gubitaka

ekvivalent

elek. linija bez gubitaka, L, C, l ← TML



⇒ Odnose napona i struja na liniji bez gubitaka na mj. x

$$\boxed{-\frac{\partial v}{\partial x} = L\frac{\partial i}{\partial t}} \qquad \boxed{-\frac{\partial i}{\partial x} = C\frac{\partial v}{\partial t}}$$

$$-\frac{\partial p}{\partial x} = \underbrace{\frac{\rho}{A}}_{L}\frac{\partial u}{\partial t} \qquad -\frac{\partial u}{\partial x} = \underbrace{\frac{A}{\rho c^2}}_{C}\frac{\partial p}{\partial t}$$

$p \ldots v$ } tlak == napon

$u \ldots i$ } protok == struja

$L = \frac{\rho}{A}$ ... akustični induktivitet

$C = \frac{A}{\rho c^2}$ ... akustični kapacitet linija bez gubitaka

$$Z_0 = \sqrt{\frac{L}{C}}$$

karakteristična impedancija

$$Z_0 = \sqrt{\frac{\frac{\rho}{A}}{\frac{A}{\rho c^2}}} = \sqrt{\frac{\rho^2 c^2}{A^2}} = \frac{\rho c}{A}$$

$$\begin{cases} u(x,t) = U(x,\Omega)\cdot e^{j\Omega t} \\ p(x,t) = P(x,\Omega)\cdot e^{j\Omega t} \end{cases}$$

$$-\frac{\partial p}{\partial x} = \frac{\rho}{A}\cdot\frac{\partial u}{\partial t}$$

$$-\frac{\partial P}{\partial x}\cdot \cancel{e^{j\Omega t}} = \frac{\rho}{A} U\cdot j\Omega\, \cancel{e^{j\Omega t}}$$

akust. impad.

$$-\frac{\partial P}{\partial x} = j\frac{\Omega\rho}{A}\cdot U \qquad Z = j\frac{\Omega\rho}{A}$$

$$-\frac{\partial u}{\partial x} = \frac{A}{\rho c^2} \cdot \frac{\partial p}{\partial t}$$

$$-\frac{\partial U}{\partial x} \cdot \cancel{e^{j\Omega t}} = \frac{A}{\rho c^2} \cdot P \cdot (j\Omega) \cdot \cancel{e^{j\Omega t}}$$

$$-\frac{\partial U}{\partial x} = j\frac{A\Omega}{\rho c^2} P \qquad Y = j\frac{A\Omega}{\rho c^2}$$

akusticna admitancija

Y

$$-\frac{\partial P}{\partial x} = z \cdot U$$

P, U funk. od x ne od t

$$-\frac{\partial U}{\partial x} = y \cdot P$$

$\Rightarrow$ parcijalne dif. jed.

$$\left.\begin{aligned} -\frac{dP}{dx} &= zU \\ -\frac{dU}{dx} &= yP \end{aligned}\right\}$$

zam. sa obicnim dif. jed.

$U(x, \Omega)$

$P(x, \Omega)$

Pretpostavljamo tj. za $P(x,\Omega)$ i $U(x,\Omega)$ koje zadovoljavaju par diferencijalnih jed.:

$$P(x,\Omega) = De^{\gamma x} + Ee^{-\gamma x}$$

$$U(x,\Omega) = Fe^{\gamma x} + Ge^{-\gamma x}$$

$$\gamma = \sqrt{Z \cdot Y} = \frac{j\Omega}{c}$$

iz rubnih uvjeta

komp. amp. izvora

$$\begin{cases} u(0,\Omega) = U_G(\Omega) \\ P(l,\Omega) = 0 \end{cases}$$

kratki spoj na kraju

nalazimo D, E, F, G

$\Rightarrow$ Uvrštavanjem nađ. konst.
dobivamo izraze za $u(x,t)$, $p(x,t)$
koji su isti kao i prije izvedeni:

Prijenosna funk. za cijev sa ulaza na izlaz. (za brzinu protoka)

$$u(0,t) = U_G \cdot e^{j\Omega t} \quad \text{(pobuda)}$$

$$u(l,t) = U(l,\Omega) U_G \cdot e^{j\Omega t} \quad \text{(odziv)}$$

Frekv. karak.:

$$V_a(j\Omega) = \frac{U(l,\Omega) \cdot \cancel{U_G}}{\cancel{U_G}} = U(l,\Omega)$$

Općenito

$$U(x,\Omega) = \frac{\cos\frac{\Omega}{c}(l-x)}{\cos\frac{\Omega}{c}\cdot l}$$

$x = l$

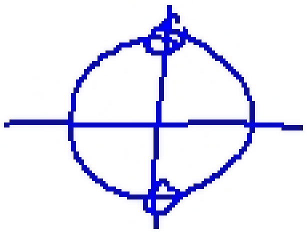
$$U(l,\Omega) = \frac{1}{\cos\frac{\Omega}{c}l}$$

$$V_a(j\Omega) = U(l,\Omega) = \frac{1}{\cos\frac{\Omega}{c}l}$$

$|V_a(j\Omega)|$

$$\left.\begin{array}{l}\text{ide u } \infty \\ \text{kada } \cos = 0\end{array}\right\} \Omega_p$$

$$\left.\begin{array}{l}\text{ide u } 1 \\ \text{kada je } |\cos| = 1\end{array}\right\} \Omega_1$$



$\cos\left(\frac{\Omega}{c} l\right)$

$$\frac{\Omega_P}{c} l = (2k+1) \cdot \frac{\pi}{2}$$

$$\Omega_p = \frac{c}{l} (2k+1) \cdot \frac{\pi}{2} \quad k \in \mathbb{Z}$$

$[rad/s]$

$$= 2\pi \cdot f_p$$

$$f_p = \frac{c}{l} \cdot (2k+1) \cdot \frac{1}{4} \quad [Hz]$$

$c = 350\, m/s$

$l = 17{,}5\, cm = 0{,}175\, m$

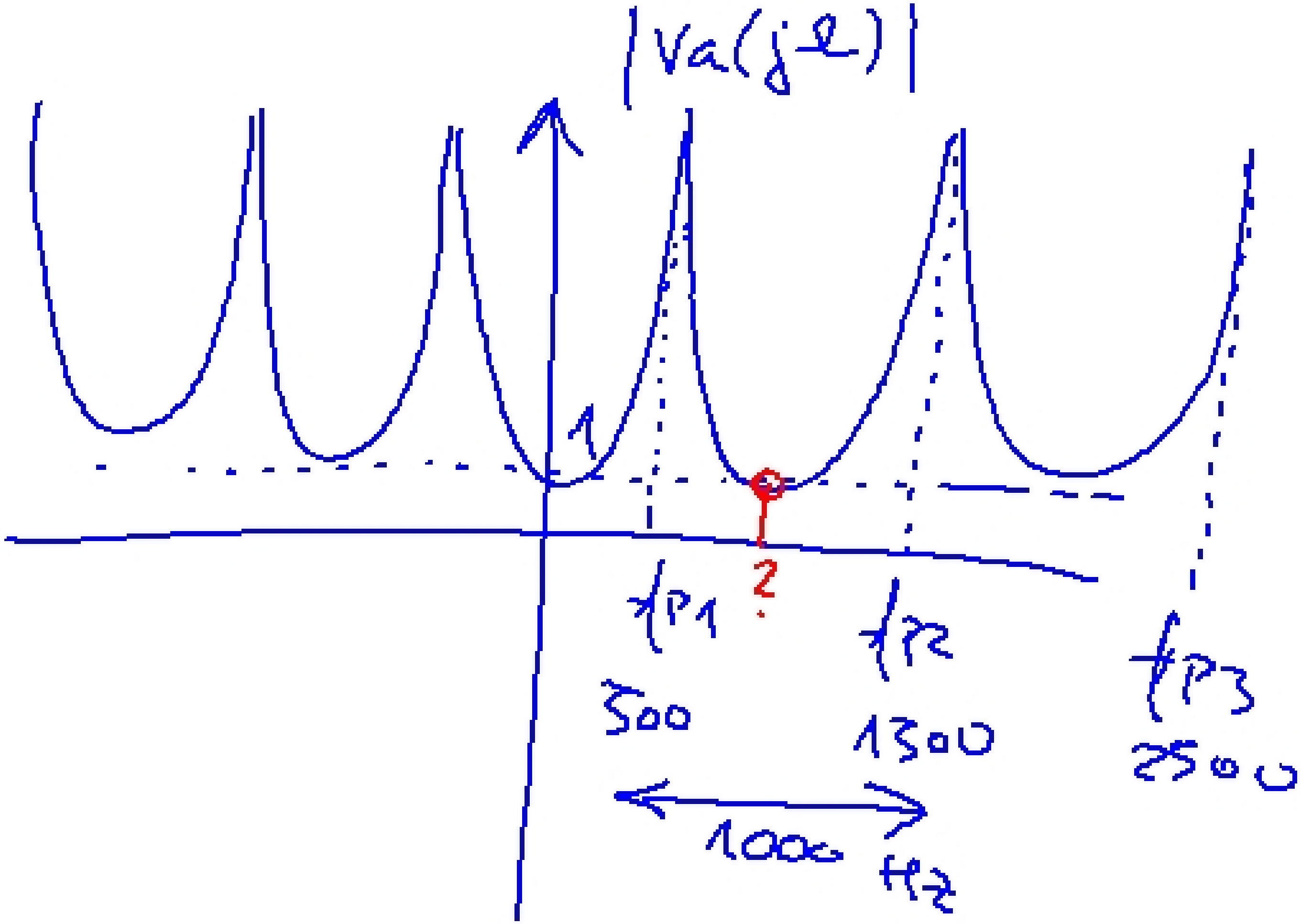
$\frac{c}{l} = 2000 \cdot s^{-1} = 2000\, Hz$

$f_p = 2000 \cdot \frac{2k+1}{4} \quad k \in Z \quad [Hz]$

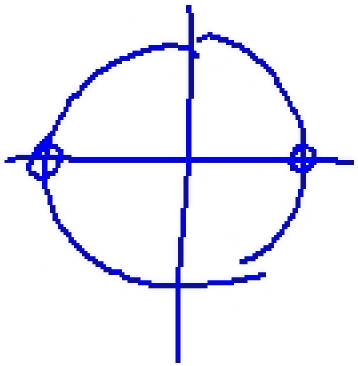
$k = 0 \quad f_p = 500\, Hz$

$k = 1 \quad f_p = 1500\, Hz$

$k = -1 \quad f_p = -500\, Hz$


|Va(jΩ)|
1
2
fP1
500
fP2
1300
fP3
2500
1000 Hz

$$\Omega_1 = \frac{c}{l}\cdot k\pi \qquad k\in \mathbb{Z}$$



$$f_1 = \frac{c}{l}\cdot\frac{k\cancel{\pi}}{2\cancel{\pi}}$$

$$= \frac{c}{l}\,\frac{k}{2} = 1000\cdot k \ [\mathrm{Hz}]$$

$$k\in \mathbb{Z}$$

f_1 $\quad 0,\ \pm 1000,\ \pm 2000,\ \dots$

$Va(s)$?

$$u(x,t) = \frac{e^{j\frac{\Omega}{c}(2l-x)} + e^{j\frac{\Omega}{c}x}}{1 + e^{j\frac{\Omega}{c}\cdot 2l}} \, U_G(\Omega)\cdot e^{j\Omega t}$$

$Va(j\Omega)$

za $x = l$

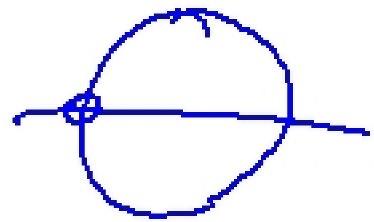
$j\Omega \to s$

$$Va(j\Omega) = \frac{2\cdot e^{j\frac{\Omega}{c}\cdot l}}{1 + e^{j\frac{\Omega}{c}2l}}$$

$$Va(s) = \frac{2\cdot e^{\frac{s}{c}\cdot l}}{1 + e^{\frac{s}{c}\cdot 2l}}$$

$V_a(s) = \infty$?

za koji s

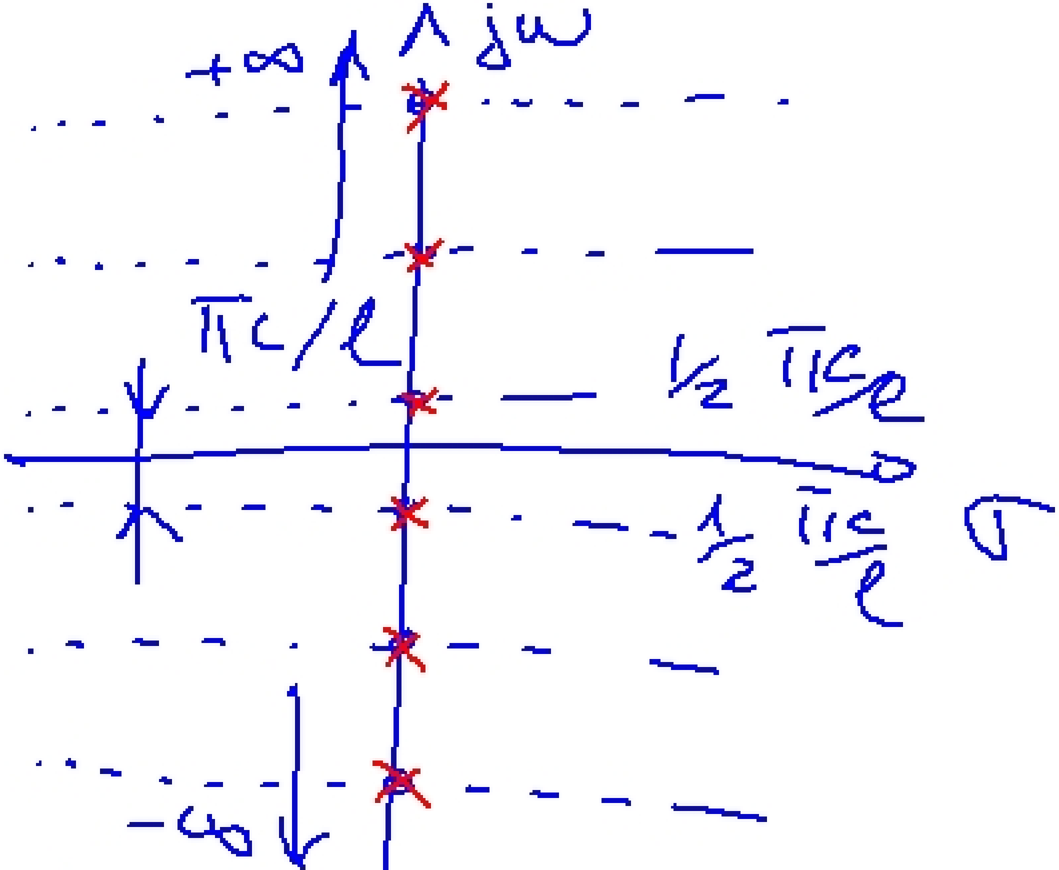


$$1 + e^{\frac{s}{c}\cdot 2l} = 0$$

$$e^{\frac{s}{c}\cdot 2l} = -1 = e^{j(2k+1)\pi} \quad k \in \mathbb{Z}$$

$$\frac{s_p}{c}\cdot 2l = j(2k+1)\pi$$

$$s_p = j\frac{(2k+1)}{2}\cdot\frac{\pi c}{l}$$



$\Rightarrow$ beskonacno mnogo polova

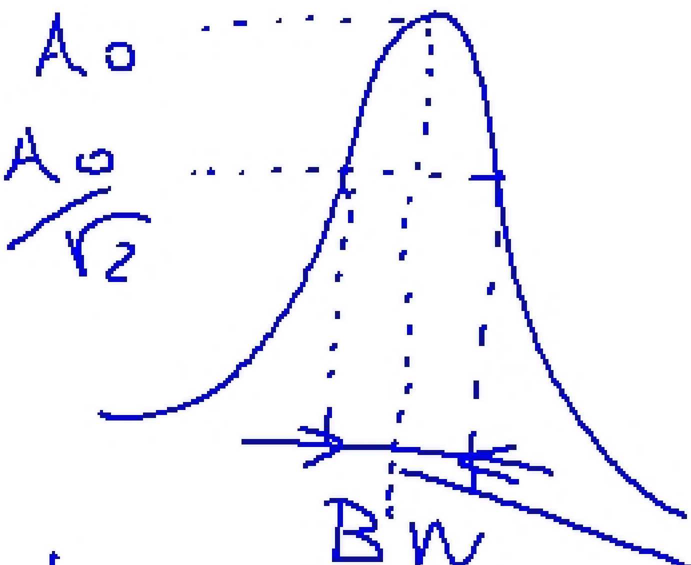
Realni model $\boxed{VT.}$

$\Rightarrow$ ... imamo gubitke

- uslijed toplinske vodljivosti
- uslijed trenja
- uslijed elastičnosti stjenki VT

$\Rightarrow$

Širina pojasa



bandwidth
širina pojasa

f_0 centralna frekv. rezonatora