$\boxed{\text{kov.}}$

$$E_n = \sum_{m=\emptyset}^{N-1} e_n^2(m)$$

$$\Phi_n(i,k) = \sum_{m=\emptyset}^{N-1} s_n(m-i)\, s_n(m-k)$$

$$\Downarrow$$

$$\begin{bmatrix} \Phi_n(1,1) & \dots & \Phi_n(1,p) \\ \Phi_n(p,1) & \dots & \Phi_n(p,p) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} \Phi(1,\emptyset) \\ \vdots \\ \Phi(p,\emptyset) \end{bmatrix}$$

$k \in [\emptyset, P] \qquad i \in [1, P]$

$$\sum_{k=1}^{P} \alpha_k \Phi_y(i,k) = \Phi_y(i,\emptyset)$$

$i = 1, 2 \ldots P$

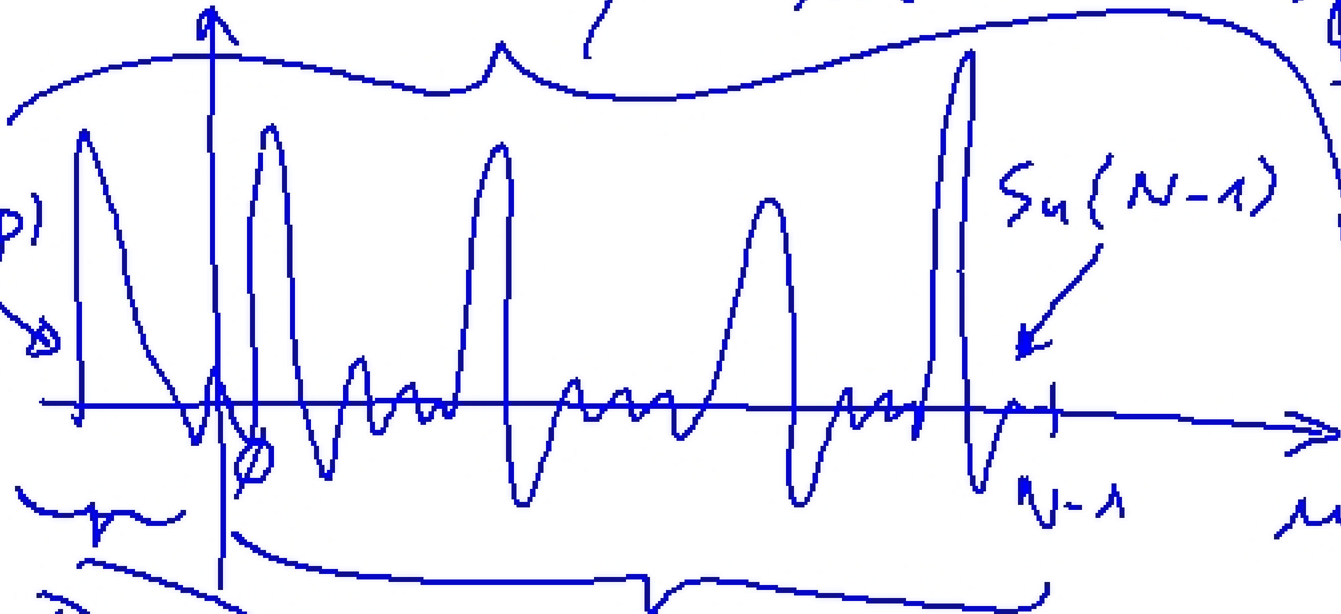
$$\Phi_y(i,k) = \sum_{m=\emptyset}^{N-1} s_y(\underbrace{m-i}_{m'})\, s_y(m-k)$$

$m = m' + i$

$$\Phi_y(i,k) = \sum_{m'=-i}^{N-1-i} s_y(m')\, s_y(m' + (i-k))$$

$$\Phi_y(i,k) = \sum_{m'=-k}^{N-1-k} s_y(m')\, s_y(m' + (k-i))$$

svih N+P uzoraka sudjeluje u izračunu $\Phi_n(i,k)$ za sve komb. i,k



P uzoraka iz prošlog okvira minimizac.

$\emptyset$ ... je dio gdje se minimizira E_n

$[\emptyset \dots N-1]$

$$Su(-1)\cdot Su(-1) + \ldots + Su(N-2)\,Su(N-2)$$

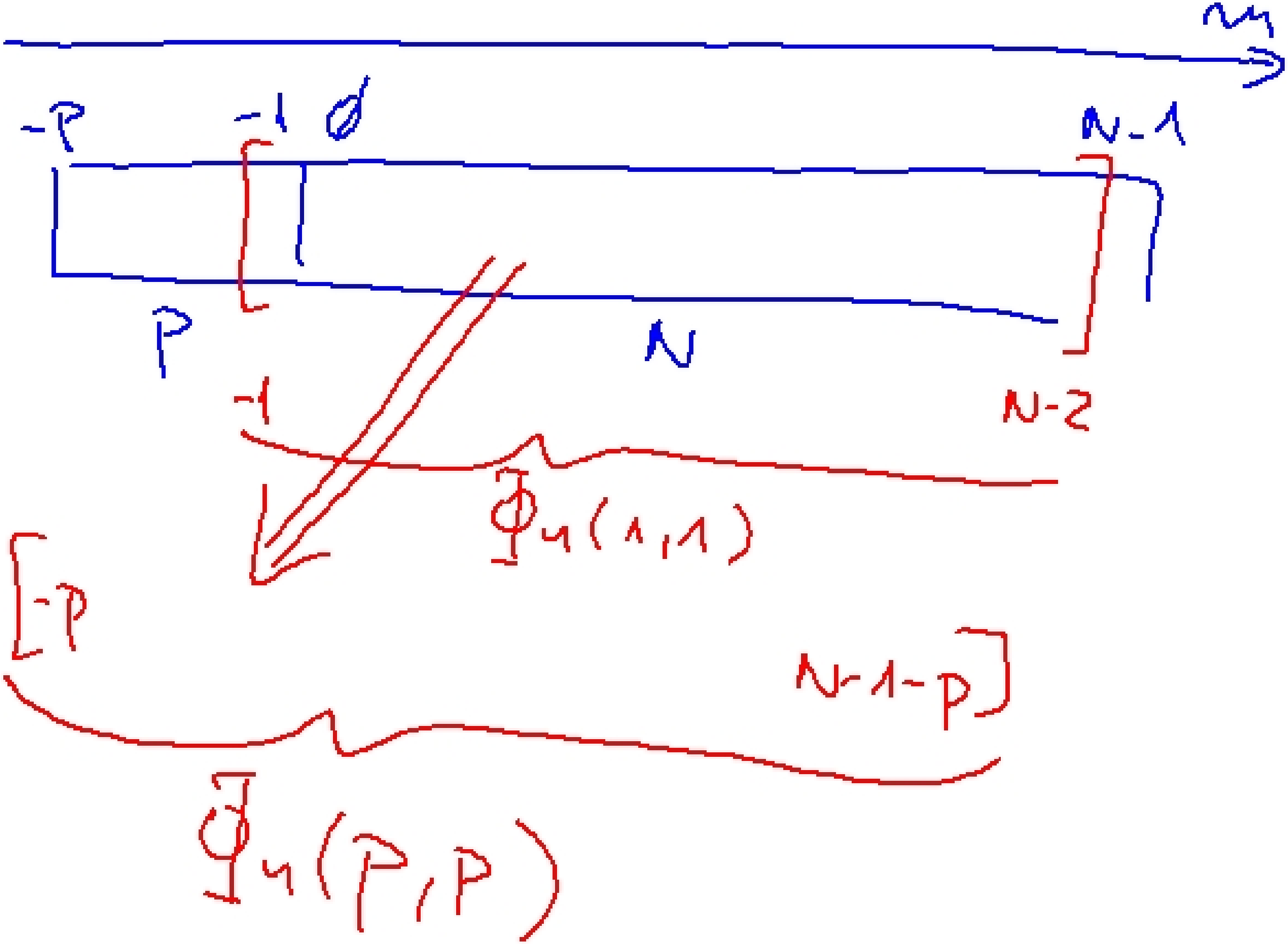
$$\begin{bmatrix} \Phi(1,1) & \Phi(1,p) \\ \Phi(p,1) & \Phi(p,p) \end{bmatrix}$$

$$Su(-1)\cdot Su(-p) + \ldots + Su(N-2)\,Su(N-p-1)$$

energije N susjednih uzoraka uz p različitih pomaka u vrem

$$Su(-p)\cdot Su(-1) + \ldots\, Su(N-p-1)\,Su(N-2)$$

$$Su(-p)\cdot Su(-p) + \ldots + Su(N-p-1)\,Su(N-p-1)$$


n
-P
-1
0
N-1
P
N
-1
N-2
Φu(1,1)
[-P
N-1-P]
Φu(P,P)

$$\Phi_u(2,1) =$$

višak u grupu

$$S_u(-2)\cdot S_u(-1) + \ldots + S_u(N-3)\, S_u(N-2)$$

$$\Phi_u(3,2) =$$

višak u ovom izrazu

$$S_u(-3)\, S_u(-2) + S_u(-2)\cdot S_u(-1) + \ldots$$

$$+ S_u(N-4)(N-3)$$

$$\Phi_u(3,2) = \Phi_u(2,1) - S_u(N-3)\,S_u(N-2) + S_u(-3)\,S_u(-2)$$

$$\Phi(i,k) \rightsquigarrow \Phi_u(i+1,k+1)$$

$$\Phi_u(i+1,k+1) = \Phi_u(i,k) - S_u(N-(i+1))\,S_u(N-(k+1)) + S_u(-i-1)\,S_u(-k-1)$$

desna strana

Donji trokutasti dio matrice sustava

simetri..

$$
\begin{bmatrix}
\Phi_u(\emptyset,\emptyset) & & & & \\
\Phi_u(1,\emptyset) & \Phi_u(1,1) & & & \\
\Phi_u(2,\emptyset) & \Phi_u(2,1) & \Phi_u(2,2) & & \\
 & \Phi_u(3,1) & \Phi_u(3,2) & & \\
 & & \vdots & & \\
\Phi_u(p-1,\emptyset) & & & & \\
\Phi_u(p,\emptyset) & \Phi_u(p,1) & \Phi_u(p,2) & \cdots & \Phi(p,p)
\end{bmatrix}
$$

za odred. svih elemenata sustava

naći: $\Phi_4(i, \emptyset)$ za $i \in [\emptyset, P]$ direktni izraz

na osnovu $\Phi_4(\emptyset, \emptyset) \ldots \Phi_4(P-1, \emptyset)$

odredi: $\underbrace{\Phi_4(1, 1) \ldots \Phi_4(P, 1)}_{\text{prvi stupac mat.}}$

na osnovu 1. st odredi 2. st

itd.

rekurzivni izraz

Broj operacija za elemente koji se određuju direktnim izrazom:

$$(P+1)\,(N \text{ množ} + (N-1) \text{ zbrajanja})$$

Za svaki preostali element rekurzivni izraz traži

$$B = A - (0 \cdot 0) + (0 \cdot 0)$$

2 z

2 M

takvih ima $\sim P^2/2$

Rješ. sustav

$$\Phi \cdot \alpha = \Psi$$ ← slob. stup.

mat. sustava

Φ ... simetr. pozit. semi definitna mat

sve vlast. vr. ≥ 0

→ proizlazi iz strukture njenih elem.

Za takve sustave

⇒ može se koristiti postupak metoda Choleskey dekompozicije ($\sqrt{\ }$)

Matrica Φ se rastavlja na produkt tri matrice:

$$\Phi = V D V^T$$

V donja trokutasta matrica sa jediničnom dijagonalom

$$4 \; p+1 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ V_{21} & 1 & 0 & 0 \\ V_{31} & V_{32} & 1 & 0 \\ V_{41} & V_{42} & V_{43} & 1 \end{bmatrix}$$

D je dijagonalna matrica

$$\begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_4 \end{bmatrix}$$

$$\Phi \alpha = \Psi$$

$$V \cdot \underbrace{D V^T \alpha}_{\text{pomožni vektor } y} = \Psi$$

$$V \cdot y = \Psi$$

$\Rightarrow$ odredim y

rešim:

$$D V^T \alpha = y$$

$$V^T \cdot \alpha = \underbrace{D^{-1} y}$$

inverzija dij. mat

$$\begin{bmatrix} y_1/d_1 \\ \vdots \\ y_p/d_p \end{bmatrix}$$

$$\begin{bmatrix} 1/d_1 & & \\ & \ddots & \\ & & 1/d_p \end{bmatrix}$$

$$\begin{bmatrix} 1 & & & \\ V_{21} & 1 & & \\ V_{31} & V_{32} & 1 & \\ V_{41} & V_{42} & V_{43} & 1 \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ \vdots \\ \vdots \\ Y_4 \end{bmatrix} = \begin{bmatrix} \Psi_1 \\ \vdots \\ \vdots \\ \Psi_4 \end{bmatrix}$$

$V \cdot Y = \Psi$
lako $\forall j$

$Y_1 = \Psi_1$

$V_{21} \cdot Y_1 + Y_2 = \Psi_2$ $\quad Y_2 = \Psi_2 - V_{21} Y_1$

$V_{31} Y_1 + V_{32} Y_2 + Y_3 = \Psi_3 \} \Rightarrow Y_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ v_{21} & 1 & 0 \\ v_{31} & v_{32} & 1 \end{bmatrix} \begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{bmatrix} \begin{bmatrix} 1 & v_{21} & v_{31} \\ 0 & 1 & v_{32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} d_1 & 0 & 0 \\ d_1 v_{21} & d_2 & 0 \\ d_1 v_{31} & d_2 v_{32} & d_3 \end{bmatrix} \begin{bmatrix} & & \\ & -\text{||}- & \\ & & \end{bmatrix}$$

$$\Phi = \begin{bmatrix} d_1 & d_1 v_{21} & d_1 v_{31} \\ d_1 v_{21} & d_1 v_{21}^2 + d_2 & d_1 v_{21} v_{31} + d_2 v_{32} \\ d_1 v_{31} & d_1 v_{31} v_{21} + d_2 v_{32} & d_1 v_{31}^2 + d_2 v_{32}^2 + d_3 \end{bmatrix}$$

$$\Phi_u(i,k) = \sum_{j=1}^{k} V_{ik} d_j V_{kj}$$

$$1 \leq k \leq i-1$$

$$= \overbrace{V_{ik} \cdot d_k \cdot \underbrace{V_{kk}}_{=1}}^{\text{zadnji član sume}} + \sum_{j=1}^{k-1} V_{ik} d_j V_{kj}$$

dijag. el. matrice V

$$\boxed{V_{ik} \cdot d_k = \Phi_u(i,k) - \sum_{j=1}^{k-1} V_{ik} d_j V_{kj} \quad \text{za } 1 \leq k \leq i-1}$$

def. A

za dijag. el. mat. Φ

$$\Phi_n(i,i) = \sum_{j=1}^{i} V_{ij}\, d_j \cdot V_{ij}$$

$$= \underbrace{V_{ii}}_{=1}\, d_i\, \underbrace{V_{ii}}_{=1} + \sum_{j=1}^{i-1} V_{ij}\, d_j\, V_{ij}$$

izraz B

$$d_i = \Phi_n(i,i) - \sum_{j=1}^{i-1} V_{ij}^2\, d_j \qquad i = 1 \ldots p$$

spec. za $i=1$ $\quad d_1 = \Phi_n(1,1)$

1. izraz B ... odredim d_1

$$d_1 = \Phi_u(1,1)$$

2. izraz A ... $V_{i,1}$ za $i = 2 \dots P$

$k=1$

$$V_{i,1} \cdot d_1 = \Phi_u(i,1) - \underbrace{\sum_{j=1}^{k-1} V_{ik} \, d_j \, V_{kj}}_{\text{ne postoji}}$$

$$V_{i,1} = \Phi_u(i,1) / d_1$$

$$V_{2,1} = \Phi_u(2,1) / d_1$$

$$V_{31} = \Phi_u(3,1) / d_1$$

iz prvog stupca mat. Φ

i odredenog elementa d_1

odredujemo 1. st. mat. V

$$\begin{bmatrix} 1 & & \\ v_{21} & 1 & \\ v_{31} & v_{32} & \\ \vdots & \vdots & \ddots \\ v_{p1} & v_{p2} & \end{bmatrix}$$

II

IV

d_1

I

d_2

III

$\ddots$

pomoću izraza B
odredujemo d_2

$$d_2 = \Phi_{22} - V_{21}^2 d_1$$

znamo

$\Rightarrow$ primjenom izraza A
odredujemo V_{i2} $i = k+1 .. p$

$$V_{3,2} \cdot \underbrace{d_2}_{\text{imam}} = \Phi_{32} - \underbrace{V_{31} d_1 \cdot V_{21}}_{\text{imam}}$$

$\Rightarrow V_{32}$

$$E_u = \Phi_u(\phi,\psi) - \sum_{k=1}^{P} \alpha_k \cdot \Phi_u(k,\phi)$$

$$E_u = \Phi_u(\phi,\psi) - \alpha^T \cdot \Psi$$

$$D \cdot V^T \cdot \alpha = Y \qquad \text{II system.}$$

$$V^T \cdot \alpha = D^{-1} \cdot Y \quad /^T$$

$$\alpha^T \cdot V = Y^T \cdot \underbrace{(D^{-1})^T}_{= D^{-1}}$$

$$\alpha^T \cdot V = Y^T \cdot D^{-1}$$

$$\alpha^T = Y^T \cdot D^{-1} \cdot V^{-1}$$

$$E_n = \Phi_n(\phi,\phi) - \underbrace{\alpha^T \cdot \Psi} = \underbrace{\Phi_n(\phi,\phi) - \sum_{i=1}^{p} \frac{y_i^2}{d_i}}_{\text{vrijedi } \nabla \text{ za sve u niže red.}}$$

$$-y^T D^{-1} \cdot \underbrace{V^{-1} \cdot \Psi}_{?}$$

1. sustav jed.

$$V \cdot y = \Psi$$

$$y = \underbrace{V^{-1} \cdot \Psi}$$

$$= -y^T \cdot D^{-1} \cdot y$$

$$[y_1 \dots y_p] \begin{bmatrix} d_1^{-1} & & \\ & \ddots & \\ & & d_p^{-1} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

$$\frac{y_1^2}{d_1} + \frac{y_2^2}{d_2} \dots \frac{y_p^2}{d_p}$$